8.4a Improper Integrals

Improper Integrals with Infinite Limits

$$\int_0^\infty e^{-\frac{x}{2}}\,dx = \lim_{b\to\infty}\int_0^b e^{-x/2}\,dx$$

$$= \lim_{b\to\infty}\left(-2e^{-x/2}\Big|_0^b\right)$$

$$= \lim_{b\to\infty}\left(-2e^{-b/2} - -2e^{0/2}\right)$$

$$\lim_{b\to\infty}\left(\frac{-2}{e^{b/2}} + 2\right) = 2$$

$$\int_0^\infty e^{-x/2}\,dx = 2$$

converges to 2

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$$\int_1^\infty \frac{1}{x}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{x}\,dx$$

diverges $= \lim_{b\to\infty} \ln x\Big|_1^b$

$= \lim_{b\to\infty} \ln b - \ln 1 = \infty$

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$$\int_1^\infty xe^{-x}\,dx = \lim_{b\to\infty}\int_1^b xe^{-x}\,dx$$

converges

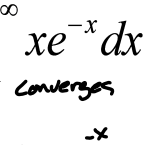
$= \lim_{b\to\infty}\left(-xe^{-x} - e^{-x}\Big|_1^b\right)$

$= \lim_{b\to\infty}\left(\left[-be^{-b} - e^{-b}\right] - \left[-e^{-1} - e^{-1}\right]\right)$

$\lim_{b\to\infty} \frac{-b}{e^b} - \frac{1}{e^b} - \left(-\frac{1}{e} - \frac{1}{e}\right)$

$\lim_{b\to\infty} \frac{-1}{e^b} \to 0$ $\quad \frac{\infty}{\infty}$ $\quad 0$ $\quad 0$

$\frac{2}{e}$



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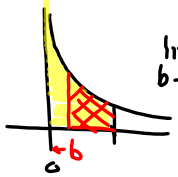
Integrals with infinite discontinuities

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx = \lim_{b\to 0^+}\int_b^1 \frac{1}{\sqrt{x}}\,dx$$

$\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

$\lim_{b\to 0^+}\left(2x^{1/2}\Big|_b^1\right)$

$\lim_{b\to 0}\left(2 - 2\sqrt{b}\right) = 2$



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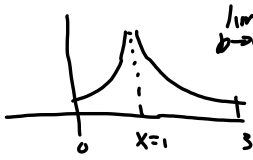
Infinite discontinuities at an interior point

$$\int_0^3 \frac{1}{(x-1)^{2/3}}\,dx = \int_0^1 \frac{1}{(x-1)^{2/3}}\,dx + \int_1^3 \frac{1}{(x-1)^{2/3}}\,dx$$



$$\lim_{b\to 0}\int_0^b (x-1)^{-2/3}\,dx$$

$$\lim_{b\to 1} 3(x-1)^{1/3}\Big|_0^b$$

$$\lim_{b\to 1} 3(b-1)^{1/3} - 3(0-1)^{1/3} = 3$$

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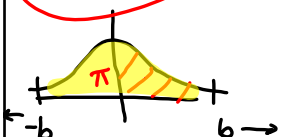
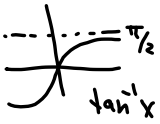
$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \pi = 2\int_0^{\infty} \frac{1}{1+x^2}\,dx$$



$$\lim_{b\to\infty} 2\int_0^b \frac{1}{1+x^2}\,dx$$

$$\lim_{b\to\infty} 2\tan^{-1}x\Big|_0^b$$

$$\lim_{b\to\infty} 2\tan^{-1}b - 2\tan^{-1}0$$



$$2\cdot\frac{\pi}{2} - 2\cdot 0 = \pi$$

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